# NAG Fortran Library Routine Document

# G01FAF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

## 1 Purpose

G01FAF returns the deviate associated with the given probability of the standard Normal distribution, via the routine name.

## 2 Specification

```
real FUNCTION G01FAF(TAIL, P, IFAIL)
INTEGER              IFAIL
real                 P
CHARACTER*1          TAIL
```

## 3 Description

The deviate, $x_p$ associated with the lower tail probability, $p$, for the standard Normal distribution is defined as the solution to

$$P(X \leq x_p) = p = \int_{-\infty}^{x_p} Z(X)dX,$$

where

$$Z(X) = \frac{1}{\sqrt{2\pi}} e^{-X^2/2}, \quad -\infty < X < \infty.$$

The method used is an extension of that of Beasley and Springer (1977). $p$ is first replaced by $q = p - 0.5$.

(a) If $|q| \leq 0.3$, $x_p$ is computed by a rational Chebyshev approximation

$$x_p = s\frac{A(s^2)}{B(s^2)}$$

where $s = \sqrt{2\pi}\, q$ and $A$, $B$ are polynomials of degree 7.

(b) If $0.3 < |q| \leq 0.42$, $x_p$ is computed by a rational Chebyshev approximation

$$x_p = \text{sign}\, q\left(\frac{C(t)}{D(t)}\right),$$

where $t = |q| - 0.3$ and $C$, $D$ are polynomials of degree 5.

(c) If $|q| > 0.42$, $x_p$ is computed as

$$x_p = \text{sign}\, q\left[\left(\frac{E(u)}{F(u)}\right) + u\right],$$

where $u = \sqrt{-2 \times \log(\min(p, 1-p))}$ and $E$, $F$ are polynomials of degree 6.

For the upper tail probability $-x_p$ is returned, while for the two-tail probabilities the value $x_{p^*}$ is returned, where $p^*$ is the required tail probability computed from the input value of $p$.

## 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Beasley J D and Springer S G (1977) Algorithm AS111. The percentage points of the normal distribution *Appl. Statist.* **26** 118–120

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

## 5 Parameters

1: TAIL – CHARACTER*1 *Input*

*On entry*: indicates which tail the supplied probability represents.

If TAIL = 'L', the lower probability, i.e., $\mathrm{P}(X \leq x_p)$.

If TAIL = 'U' the upper probability, i.e., $\mathrm{P}(X \geq x_p)$.

If TAIL = 'S' the two-tail (significance level) probability, i.e., $\mathrm{P}(X \geq |x_p|) + \mathrm{P}(X \leq -|x_p|)$.

If TAIL = 'C' the two-tail (confidence interval) probability, i.e., $\mathrm{P}(X \leq |x_p|) - P(X \leq -|x_p|)$.

*Constraint*: TAIL = 'L', 'U', 'S' or 'C'.

2: P – ***real*** *Input*

*On entry*: the probability, $p$, from the standard Normal distribution as defined by TAIL.

*Constraint*: $0.0 < \mathrm{P} < 1.0$.

3: IFAIL – INTEGER *Input/Output*

*On entry*: IFAIL must be set to $0$, $-1$ or $1$. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

*On exit*: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value $-1$ or $1$ is recommended. If the output of error messages is undesirable, then the value $1$ is recommended. Otherwise, for users not familiar with this parameter the recommended value is $0$. **When the value $-1$ or $1$ is used it is essential to test the value of IFAIL on exit.**

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or $-1$, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If on exit IFAIL $\neq 0$, then G01FAF returns 0.0.

IFAIL = 1

On entry, TAIL $\neq$ 'L', 'U', 'S' or 'C'.

IFAIL = 2

On entry, $\mathrm{P} \leq 0.0$,
or $\mathrm{P} \geq 1.0$.

## 7 Accuracy

The accuracy is mainly limited by the ***machine precision***.

# 8 Further Comments

None.

# 9 Example

Four values of TAIL and P are input and the deviates calculated and printed.

## 9.1 Program Text

**Note:** the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01FAF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             DEV, P
      INTEGER          I, IFAIL
      CHARACTER        TAIL
*     .. External Functions ..
      real             G01FAF
      EXTERNAL         G01FAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01FAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) ' Tail    Probability     Deviate '
      WRITE (NOUT,*)
      DO 20 I = 1, 4
         READ (NIN,*) TAIL, P
         IFAIL = 0
*
         DEV = G01FAF(TAIL,P,IFAIL)
*
         WRITE (NOUT,99999) TAIL, P, DEV
   20 CONTINUE
      STOP
*
99999 FORMAT (3X,A1,9X,F5.3,9X,F6.4)
      END
```

## 9.2 Program Data

```
G01FAF Example Program Data
 'L' 0.975
 'U' 0.025
 'C' 0.95
 'S' 0.05
```

## 9.3 Program Results

```
 G01FAF Example Program Results

  Tail    Probability     Deviate

   L         0.975         1.9600
   U         0.025         1.9600
   C         0.950         1.9600
   S         0.050         1.9600
```